OVERVIEW OF PROPAGATION THEORY[1]

Free Space Propagation

Radio Horizon and Propagation Modes

Line of Sight and Diffraction Modes

[1] Adapted from L. E. Miller, "Propagation Model Sensitivity Study," J. S. Lee Associates, Inc. contract report , July 1992. (DTIC accession number AD-B166479)

Free-Space Propagation

- A sinusoidal waveform emitted by a point source would propagate radially in a vacuum (free space) and for that reason we can call the source an isotropic radiator.

- If the emitted power is P_{rad} W, at a distance d meters from the source, the magnitude of the wave's Poynting vector (power per unit area) is

$$\mathrm{P}_{fs} = \frac{\mathrm{P}_{rad}}{4\pi d^2} \quad \text{in W/m}^2$$

- For an antenna (radiator) that is not isotropic, at a distance large compared with the size of the antenna, the radiated power P_{rad} may be replaced by $\mathrm{P}_t\mathrm{G}_t$, where

$\mathrm{P}_t =$ power delivered to the transmitter antenna

$\mathrm{G}_t =$ transmitter antenna gain

Free-space propagation (continued)

- For derivation of the effects of Earth on radio wave propagation, it is sometimes more convenient to speak in terms of the root-mean-square electric field intensity (in volts per meter) given by

$$E_{fs} = \sqrt{Z_{fs} \mathsf{P}_{fs}}$$

where Z_{fs} , the impedance of free space, is given by

$$Z_{fs} = \sqrt{\mu_{fs}/\epsilon_{fs}} \approx 120\pi \doteq 377\,\Omega$$

$$\mu_{fs} = 4\pi \times 10^{-7}\,\text{H/m} = 4\pi \times 10^{-7}\,\text{V-s/A-m}$$

$$= \text{permeability of free space}$$

$$\epsilon_{fs} = (10^{-9}/36\pi)\,\text{F/m} = (10^{-9}/36\pi)\,\text{A-sec/V-m}$$

$$= \text{permittivity (dielectric constant) of free space.}$$

Free-space propagation (continued)

- At a relatively large distance from a nonisotropic radiator, the electric field intensity in free space is

$$E_{fs} = \sqrt{120\pi \cdot \frac{\mathsf{P}_t \mathsf{G}_t}{4\pi d^2}} = \frac{\sqrt{30\,\mathsf{P}_t \mathsf{G}_t}}{d} \quad \text{V/m}$$

- Whether or not there is free-space propagation, if the electric field intensity and power at a receiving antenna are E_{rec} and P_{rec}, respectively, the maximal useful power that could be intercepted by a matched receiver using an isotropic antenna is given by

$$\mathsf{P}_r = \frac{\lambda^2}{4\pi} \cdot \mathsf{P}_{rec}$$

- For a nonisotropic antenna (with antenna power gain G_r) and a matched receiver, the received power is, using $E_{rec} = \sqrt{Z_{fs}\mathsf{P}_{rec}}$:

$$\mathsf{P}_r = \frac{\lambda^2}{4\pi} \cdot \mathsf{P}_{rec}\mathsf{G}_r = \frac{\lambda^2}{4\pi} \cdot \frac{E_{rec}^2}{Z_{fs}} \cdot \mathsf{G}_r = \left(\frac{E_{rec}\lambda}{2\pi}\right)^2 \cdot \frac{\mathsf{G}_r}{120}$$

Free-space Propagation Loss

- The ratio of received power to transmitted power is found to be

$$\frac{\mathsf{P}_r}{\mathsf{P}_t} = \left(\frac{E_{rec}\lambda}{2\pi}\right)^2 \frac{\mathsf{G}_r}{120} \div \frac{(E_{fs}d)^2}{30\,\mathsf{G}_t} = \begin{cases} \left(\dfrac{\lambda}{4\pi d}\right)^2 \mathsf{G}_t\mathsf{G}_r \left(\dfrac{E_{rec}}{E_{fs}}\right)^2, \text{ general} \\ \left(\dfrac{\lambda}{4\pi d}\right)^2 \mathsf{G}_t\mathsf{G}_r, \text{ free space} \end{cases}$$

- This general relation shows how to assess the effects of propagation in terms of the received electric field intensity relative to the value in free space:

$$\text{Path loss} = -10\,\log_{10}\left(\frac{\mathsf{P}_r}{\mathsf{P}_t}\cdot\frac{1}{\mathsf{G}_t\mathsf{G}_r}\right) = 20\,\log_{10}\left(\frac{4\pi d}{\lambda}\right) + 20\,\log_{10}\left(\frac{E_{fs}}{E_{rec}}\right)$$

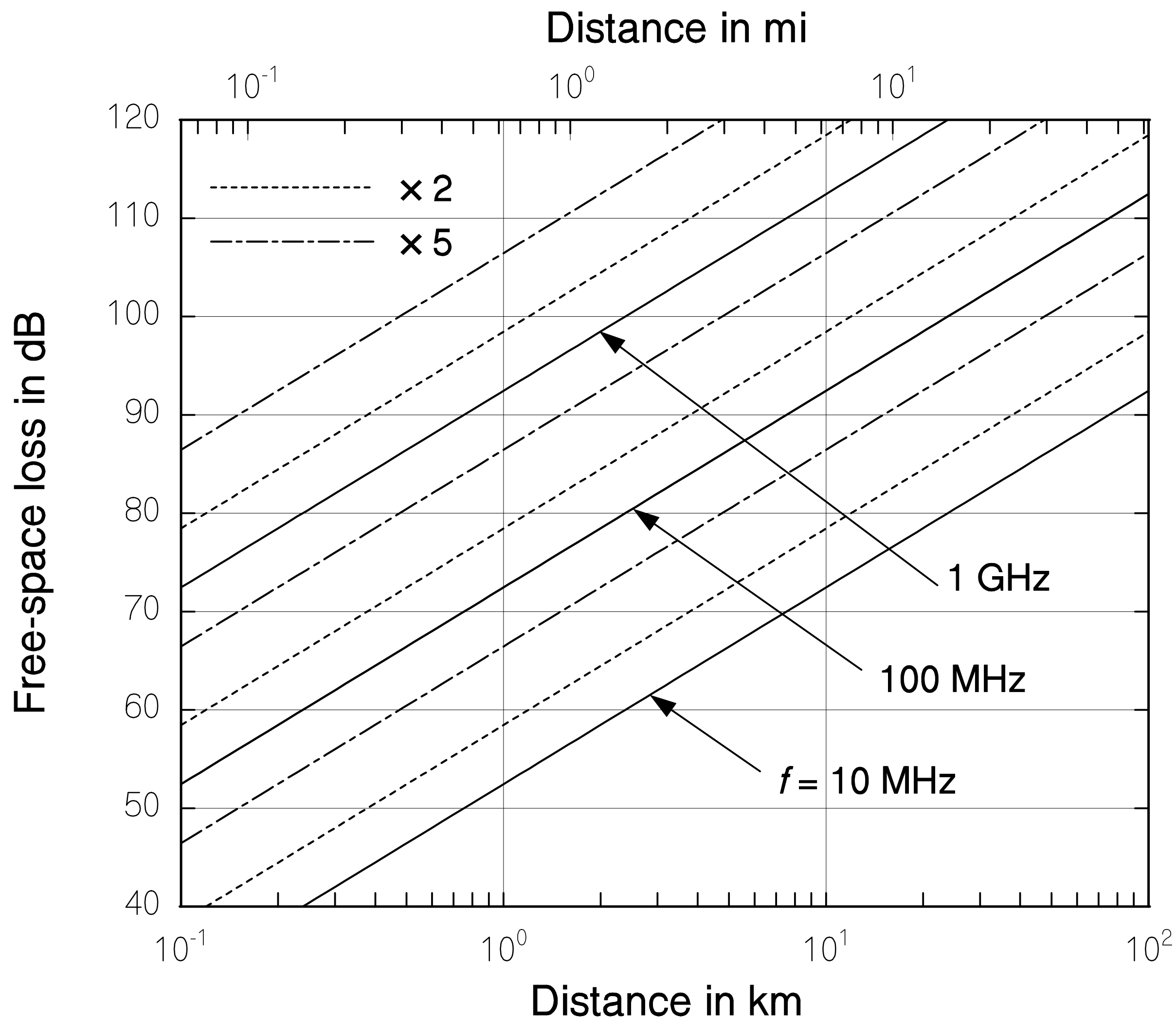
$$= L_{fs} + L_{nfs} \qquad (nfs \text{ denotes “non-free-space”})$$

and

$$L_{fs} = 20\,\log_{10}\left(\frac{4\pi d}{\lambda}\right) = 20\,\log_{10}\left(\frac{4\pi\cdot 1,000\,d_{\text{km}}}{299.8/f_{\text{MHz}}}\right)$$

$$= 32.45\,\text{dB} + 20\,\log_{10}(d_{\text{km}}\,f_{\text{MHz}}), \qquad d_{\text{km}} \equiv d \text{ in km, } f_{\text{MHz}} \equiv f \text{ in MHz}$$

$$= 36.58\,\text{dB} + 20\,\log_{10}(d_{\text{mi}}\,f_{\text{MHz}}), \qquad d_{\text{mi}} \equiv d \text{ in miles}$$



Distance in mi
10^-1
10^0
10^1
120
110
100
90
80
70
60
50
40
× 2
× 5
1 GHz
100 MHz
f = 10 MHz
10^-1
10^0
10^1
10^2
Distance in km
Free-space loss in dB

PRPTHY.6

Radio Horizon and Propagation Modes

- For ground-to-ground communications at VHF and UHF, the departure of the value of the received electric field intensity E_{rec} from the value that would occur in free space for the same distance arises chiefly from effects of the interaction of the radio wave with Earth.

- There are, however, significant effects due to the non-free-space properties of the medium of propagation itself—the air in the troposphere, that portion of the atmosphere extending to about 10 mi above Earth.

- Because of the presence of various gases in the atmosphere, including water vapor, the dielectric constant ϵ_r of the air in the troposphere is slightly greater than the value of the dielectric constant in free space (unity).

  - Because the density of these gases generally decreases with height, ϵ_r and the refractive index of the air, $n = \sqrt{\epsilon_r}$, decrease with height.

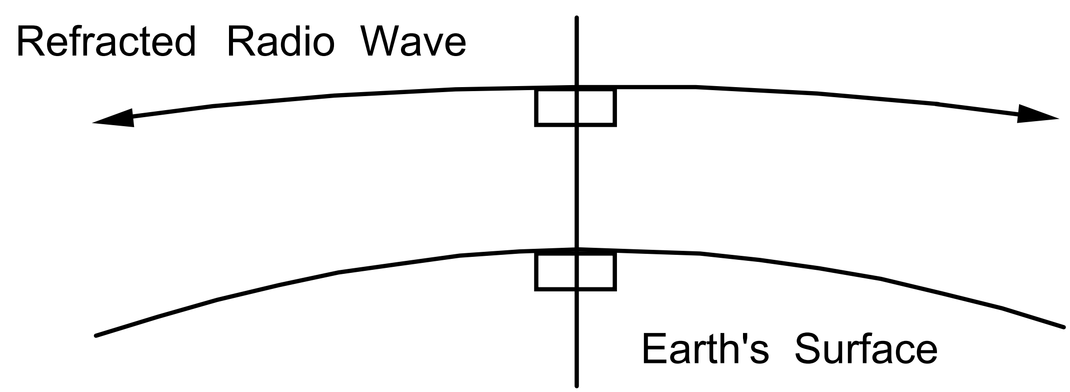
  - This variation of refractive index gives rise to propagation phenomena such as refraction, reflection, scattering, duct transmission, and fading of signals.

Curvature of Radio Waves

- Because of the decrease of the index of refraction with increase in height, the velocity of propagation increases with height in such a way that the radio wave ray paths are very nearly arcs of circles over the distances involved in ground-to-ground mobile communications.
  - The radius of curvature is $r_w \approx 4r_e$ in the standard atmosphere, where

$$r_e = \text{radius of Earth } (6{,}370\,\text{km})$$

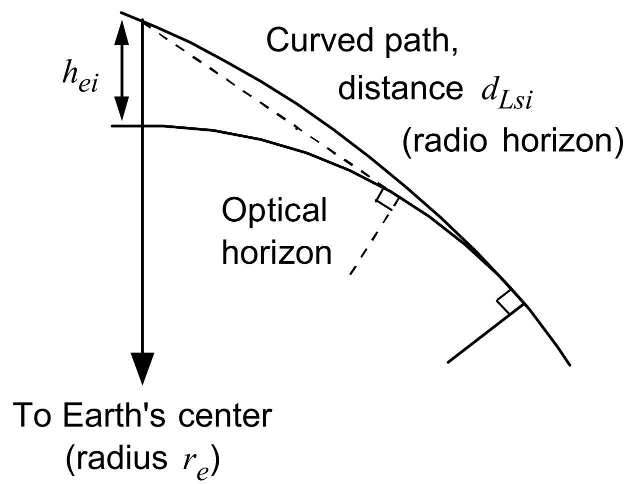
- A horizontally emitted ray is curved, bending toward Earth instead of continuing in a straight line, but not intersecting Earth (assumed to be a sphere).



Radio Horizon

- Because radio wave paths are curved, the distance to the horizon for radio signals is larger than the straight-line distance (optical path).

- The geometry of the situation in which an inclined curved ray path from or to antenna i with effective height h_{ei} is tangent to the earth is illustrated below for a "smooth Earth" with no terrain features.

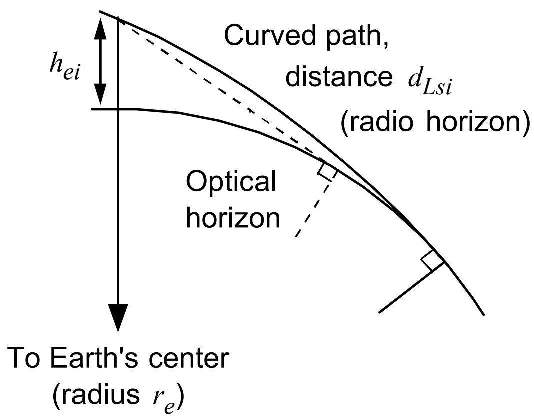


- The path distance to the tangent point is the smooth-Earth radio horizon distance for that antenna, denoted d_{Lsi} in the diagram.

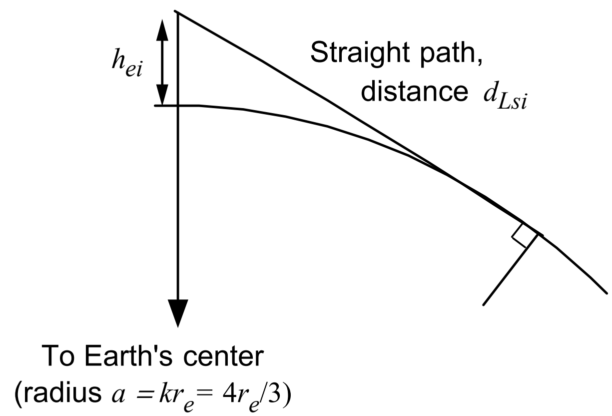
Radio horizon (continued)

- Because for analysis of communications paths it is more convenient to deal with straight-line ray paths, it is desirable to change the geometrical coordinate system so that the refracted rays appear to be straight lines.

- For this purpose, a modified tangential geometry involving a fictitious Earth, having a radius $a = kr_e > r_e$, is postulated as shown in the diagram on the right side of the diagram with the same distance d_{Lsi}.



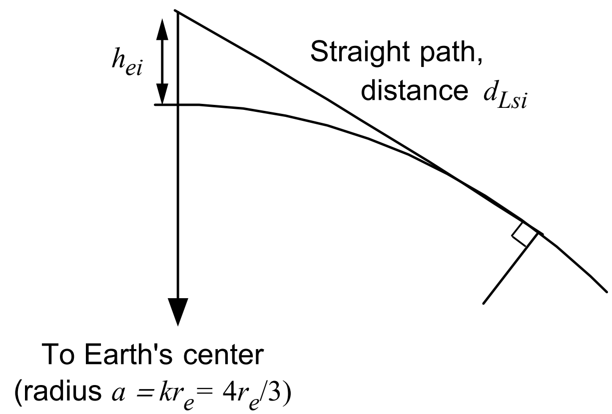
Actual horizon geometry



Fictitious horizon geometry

Radio horizon (continued)

- For the same distance of the ray path above Earth, h, for either model for any distance along the tangential ray path, it can be shown that the fictitious Earth's radius should be taken to be $a = 4r_e/3 = 8{,}493$ km.
  - The use of $a = 4r_e/3$ allows ray paths to be considered as being straight so long as they lie within the first km above Earth.



- The radio horizon distance for a smooth Earth, as a function of antenna height, is calculated as

$$d_{Lsi} = \sqrt{(a + h_{ei})^2 - a^2} \approx \sqrt{2ah_{ei}}$$

when Earth's radius and the antenna height are expressed in the same units.

Radio Horizon and Line of Sight

- For typical units, the smooth-earth radio horizon is given by

$$d_{Lsi}(\text{km}) \approx \sqrt{0.002\, a(\text{km})\, h_{ei}(\text{m})} = \sqrt{17\, h_{ei}(\text{m})}$$

$$d_{Lsi}(\text{mi}) \approx \sqrt{2\, h_{ei}(\text{ft})}$$

- A transmitter and receiver are said to be within radio line-of-sight (LOS) when the link distance d is such that

$$d_{Ls} \triangleq d_{Lst} + d_{Lsr} > d$$

- The actual radio horizon distance, as opposed to the idealized value, is typically smaller, because of the effect of terrain and buildings.

Characterization of Terrain and Its Effects

- If a hill or building is “in the way” of the tangential path depicted in the smooth-Earth model, the actual horizon distance is the distance to a tangent with that object.

- Therefore, the more hilly or populous the area is, the more likely it is that the radio horizon is closer than predicted by the smooth-Earth model.

- The presence of terrain or buildings also affects the LOS distance through the effective antenna height: if the antenna is put on top of a hill or a building, the radio horizon is much farther away.

- Typically, the altitude above sea level of the land in the vicinity of the transmitting or receiving antenna, and along the great circle surface path between them, varies to some degree above and below a “reference plane.” That is, the land altitude at a distance x along the path from transmitter to receiver can be expressed as

$$h_a(x) = h_{ref}(x) + h_s(x), \quad x = 0 \text{ (transmitter) to } d \text{ (receiver)}$$

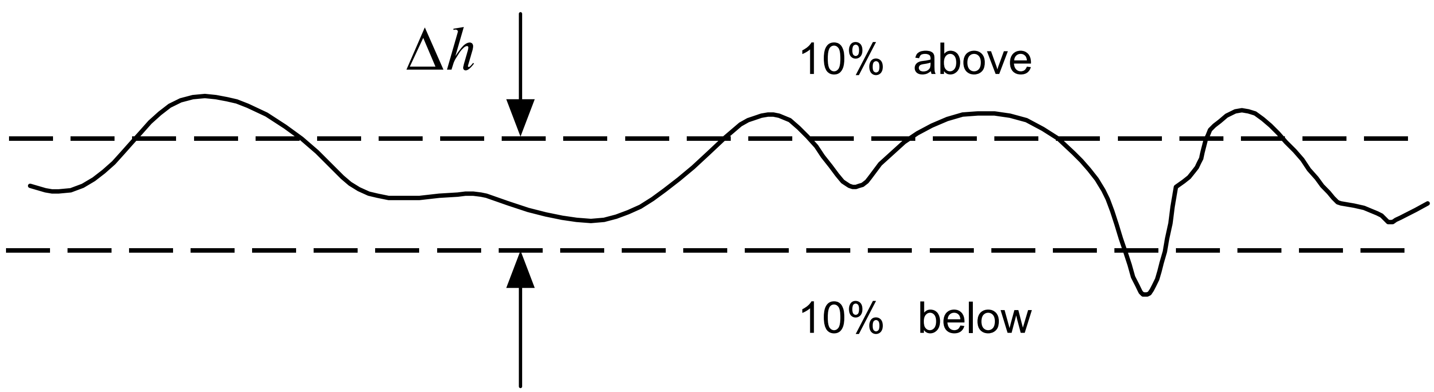
in which $h_{ref}(x)$ denotes the reference or average terrain height along the path and $h_s(x)$ denotes the variations in surface height.

Characterization of terrain (continued)

- For the purpose of modeling the effects of terrain on horizon distance and other factors affecting propagation, the reference plane can be a straight line fit to the points of $h_a(x)$ if these data are available.

- The deviations of the surface height described as samples of $h_s(x)$ are likely to have a roughly symmetrical distribution, with a zero-valued median and mean.

  - The degree of variation of the terrain can be parameterized by the "terrain irregularity parameter" Δh, the interdecile range—the difference between the value of $h_s(x)$ that is exceeded for 10% of the samples and the value that is not exceeded for 10% of the samples.



Terrain irregularity parameter

- From studies of many terrain profiles, it has been determined that the median interdecile height as a function of the length of the path d is given by

$$\Delta h(d) = \Delta h \left(1 - 0.8\, \mathrm{e}^{-d_{\mathrm{km}}/50}\right)$$

- When detailed path terrain profiles are not available, the analyst can specify a value for Δh that is chosen to fit one of the terrain descriptions in the following table:

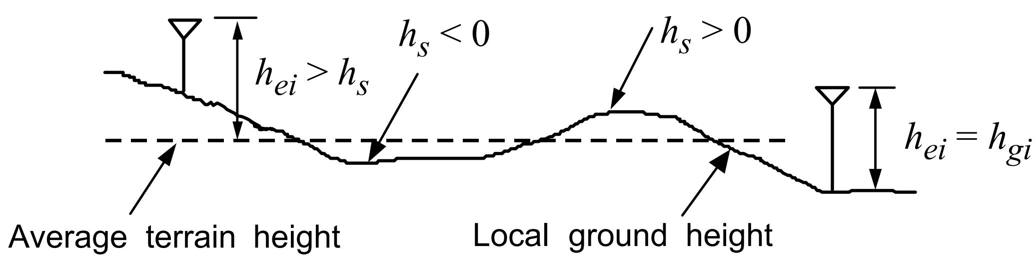
Terrain description	Δh in meters
Water or very smooth plains	0–5
Smooth plains	5–20
Slightly rolling plains	20–40
Rolling plains	40–80
Hills	80–150
Mountains	150–300
Rugged mountains	300–700
Extremely rugged mountains	> 700

Effective Antenna Height

- Let the structural height of antenna i (either $i = t$ for transmitter or $i = r$ for receiver) above the local ground be denoted h_{gi} ; then the **effective antenna height** for that antenna can be formulated as

$$h_{ei} \triangleq \max[h_{gi}\,,\, h_{gi} + h_a(x_i) - h_{ref}(x_i)]\,,\;\; i = t,\, r$$

- Let h_s denote the height of the terrain surface in a given antenna location, relative to the local average terrain height.
  - If the antenna is on a hill ($h_s > 0$), the terrain height is added to the structural height to obtain the effective antenna height; otherwise, the effective height is the actual, structural height.



h_s = Local ground height – average terrain height

h_{ei} = effective antenna height h_{gi} = structural antenna height

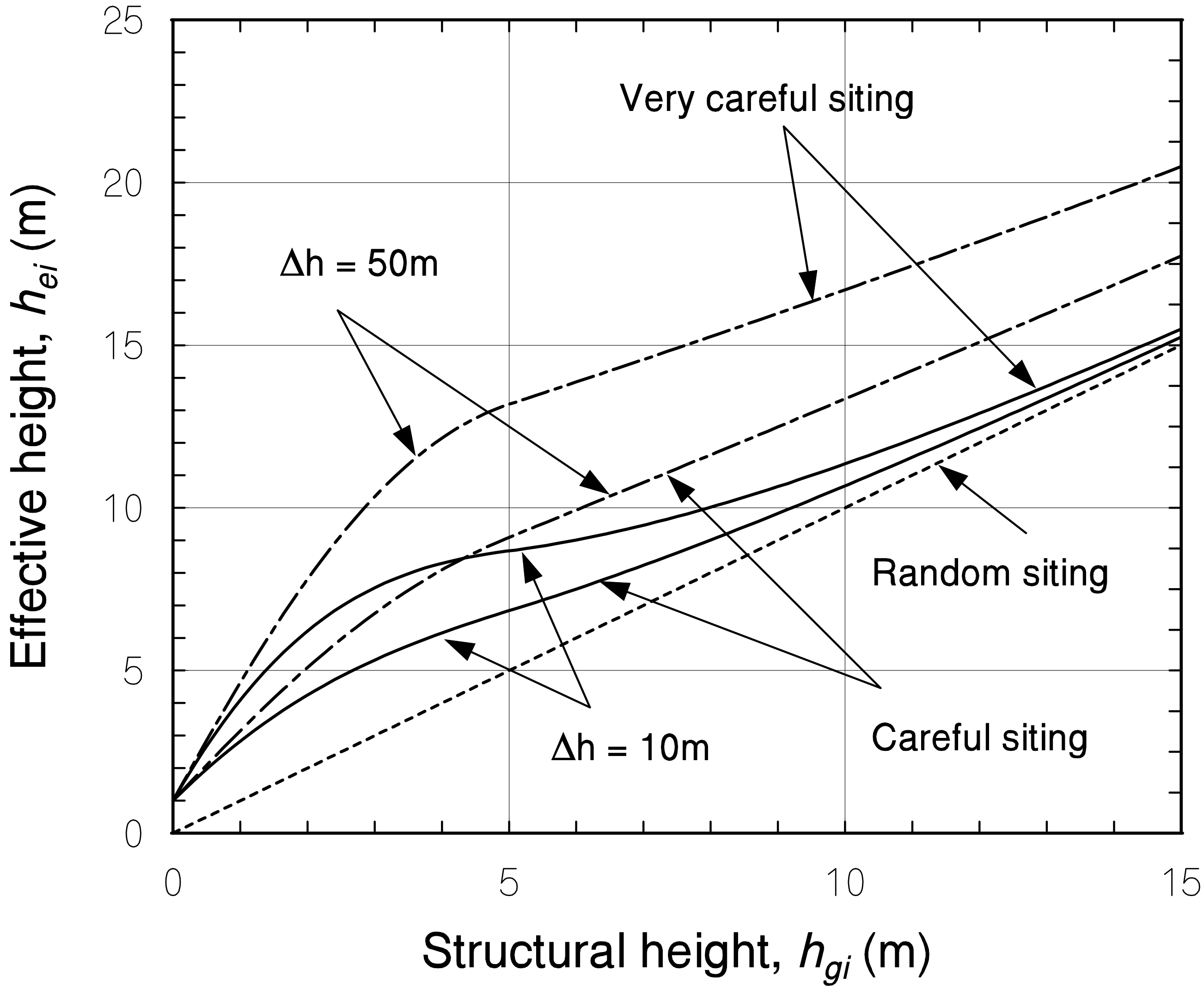
Effective antenna height (continued)

- For prediction purposes in the absence of specific terrain data, the parameter Δh can be related to a statistical treatment of the location of antenna sites.
  - If an antenna site is randomly selected, as tends to be the case for a mobile radio unit, then it is reasonable to assume that, on the average, the altitude $h_a(x)$ at the antenna site equals the reference value h_{ref}.
  - A more carefully located antenna would be placed on a hill.
- Based on the analysis of many path profiles, the following empirical formula for effective antenna height was developed for computer propagation calculations when the structural height of the antenna is 10m or less:

$$h_{ei} = \begin{cases} h_{gi}\,, & \text{random siting} \\ h_{gi} + \left[1 + c \cdot \sin\left(\frac{\pi h_{gi}}{10\,\mathrm{m}}\right)\right] \mathrm{e}^{-2h_{gi}/\Delta h}, & \text{selected siting } (h_{gi} \leq 5 \text{ m}) \\ h_{gi} + (1 + c) \cdot \mathrm{e}^{-2h_{gi}/\Delta h}, & \text{selected siting } (h_{gi} > 5 \text{ m}) \end{cases}$$

where

$$c = \begin{cases} 4, & \text{careful siting} \\ 9, & \text{very careful siting} \end{cases}$$

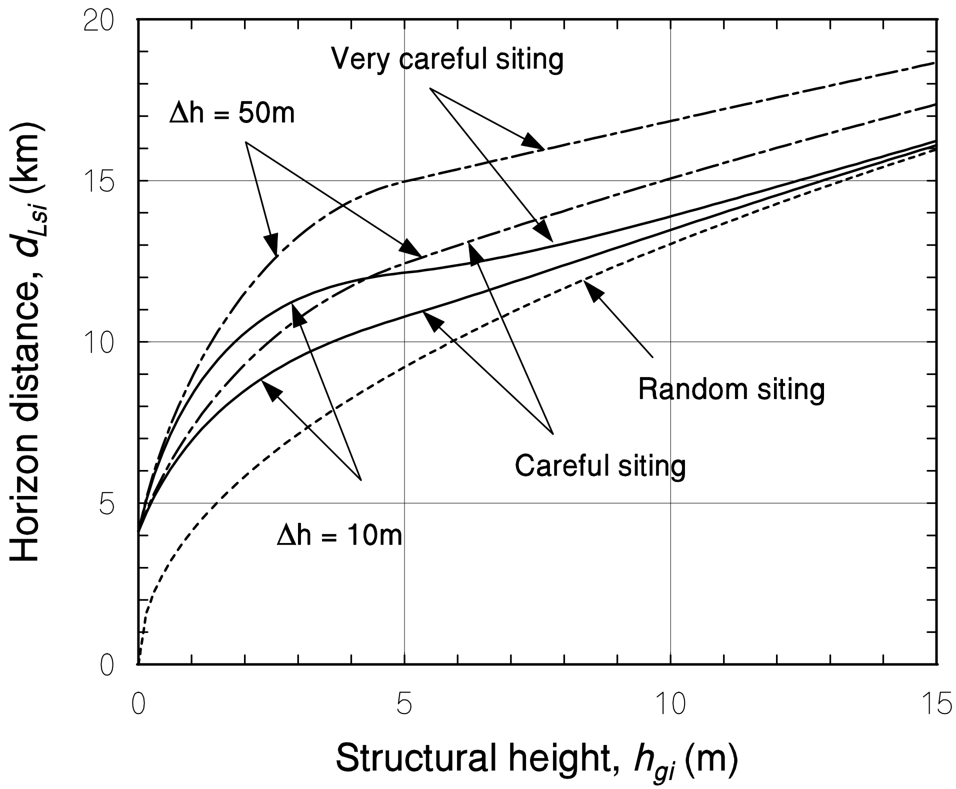


Very careful siting
Δh = 50m
Random siting
Careful siting
Δh = 10m
25
20
15
10
5
0
0
5
10
15
Effective height, h_{ei} (m)
Structural height, h_{gi} (m)

PRPTHY.18

Radio horizon using effective antenna height

- The empirical formula for effective antenna height can be used to estimate the transmitter and receiver smooth-Earth horizon distances d_{Lst} and d_{Lsr} and the LOS distance $d_{Ls} = d_{Lst} + d_{Lsr}$ using $d_{Lsi} = \sqrt{2ah_{ei}}$.



Estimate of actual horizon distance

- Because of the terrain, the actual distance to the radio horizon is less than the smooth-Earth horizon distance.
- The more irregular the terrain (the larger Δh), the smaller the likely value of the actual horizon distance.
- Median values of the actual radio horizons d_{Li} and LOS distance $d_L = d_{L1} + d_{L2}$ for irregular terrain then can be estimated from the empirical formula:

$$d_{Li} = d_{Lsi} \times \mathrm{e}^{-0.07\sqrt{\Delta h/\mathrm{max}(h_{ei},\, 5\,\mathrm{m})}}$$

Propagation Modes

- The effects of the presence of Earth on the ground-to-ground propagation of radio waves at VHF and UHF depend on path length.

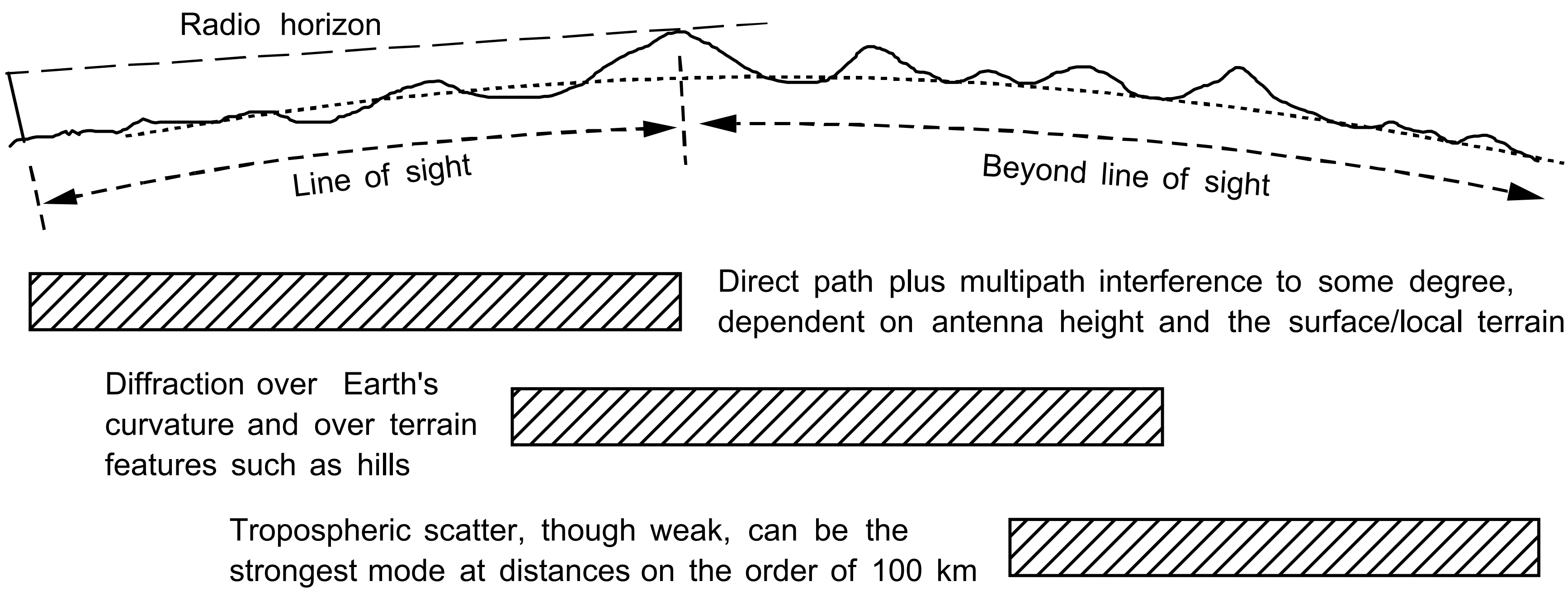
- For shorter, LOS paths, the main influence of Earth is to cause a reflected ray to arrive at the receiver in addition to the direct ray, causing destructive or constructive interference.

- For beyond-line-of-sight (BLOS) or transhorizon paths, the propagation of the signal over the horizon is possible because of either diffraction of the wave over the horizon, scattering of the signal from the troposphere, or both.

- Generally there is a gradual transition between one mode of propagation and another.

- The dominant mode of propagation is determined by the link distance d. If d is less than the combined horizon distances of the transmitter and receiver antennas, which is called the LOS distance, the dominant propagation mode is LOS.

Dominant propagation modes for ground-to-ground communications as a function of distance from the transmitter



Horizon distance for a mobile unit

- From the “random siting” curves, it is evident that a mobile unit in a cellular system, with an antenna height of about $h_{gm} = h_{em} = 2$ m, has a smooth-Earth radio horizon of about $d_{Lsm} = 5$ km.

- In a rural setting, the likelihood that there is a terrain obstacle reduces this distance according to the empirical formula given above.

- **Example:** if the terrain can be characterized as “hills” with a terrain irregularity parameter value of $\Delta h = 100$m, the actual horizon distance for the mobile is about

$$d_{Lm} = 5 \cdot \mathrm{e}^{-0.07\sqrt{100/\max(2,5)}} = 5 \cdot \mathrm{e}^{-0.07\sqrt{20}} = 3.7\,\text{km}$$

Horizon distance for a base station

- A base station in the same cellular system will typically have an antenna structural height of about $h_{gb} = 100$–200 ft or 30–60 m; for definiteness, let us assume $h_{gb} = 50$m.
  - On a smooth Earth, this antenna height corresponds to a horizon distance of about $d_{Lsb} = \sqrt{16.99\, h_{gb}} = 29.1$ km.
  - In the case of actual terrain, the base station antenna would be placed on a hill and the horizon distance without obstacles is even greater.
- **Example:** For $\Delta h = 100$ m, according to the empirical "careful siting" formula, the effective antenna height of an elevated 50m base station antenna is about

$$
\begin{aligned}
h_{eb} &= h_{gb} + (1+4) \cdot \mathrm{e}^{-2h_{gb}/\Delta h} \\
&= 50 + 5\mathrm{e}^{-2 \cdot 50/100} = 51.8\ \mathrm{m}
\end{aligned}
$$

- This effective antenna height gives a smooth-Earth horizon distance of $d_{Lsb} = 29.7$ km, which is reduced to a likely actual horizon distance of

$$d_{Lb} = 29.7 \times \mathrm{e}^{-0.07\sqrt{100/51.8}} = 27.0\ \mathrm{km}$$

Propagation modes for a cellular system

- Based on the example just considered, the total LOS distance for a base-to-mobile link or mobile-to-base link is likely to be on the order of $d_L = d_{Lb} + d_{Lm} = 27.0 + 3.7 = 30.7$ km.

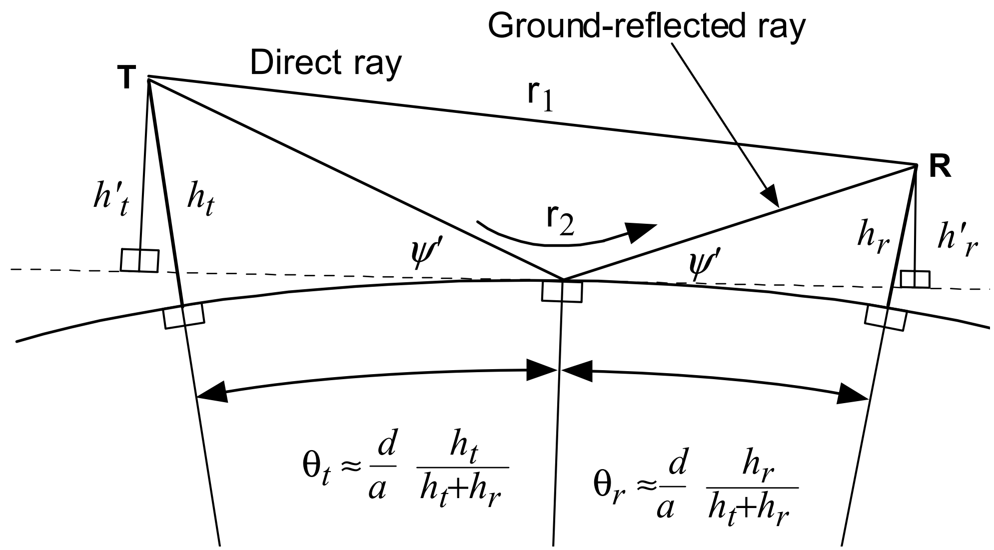
- For cellular communications, the distances involved dictate that the propagation mode that is most often involved is LOS.

- However, in an urban setting, the buildings often act as man-made terrain obstacles that reflect and block the direct LOS path between the antennas, so that the mode of propagation is almost always a complex combination of reflected paths that make their way around buildings and diffracted paths that are bent as they interact with the tops of buildings.

- For that reason it is quite difficult to predict propagation loss in urban settings using a theoretical model, and a number of empirical formulas are used for prediction.

Propagation in the LOS Region

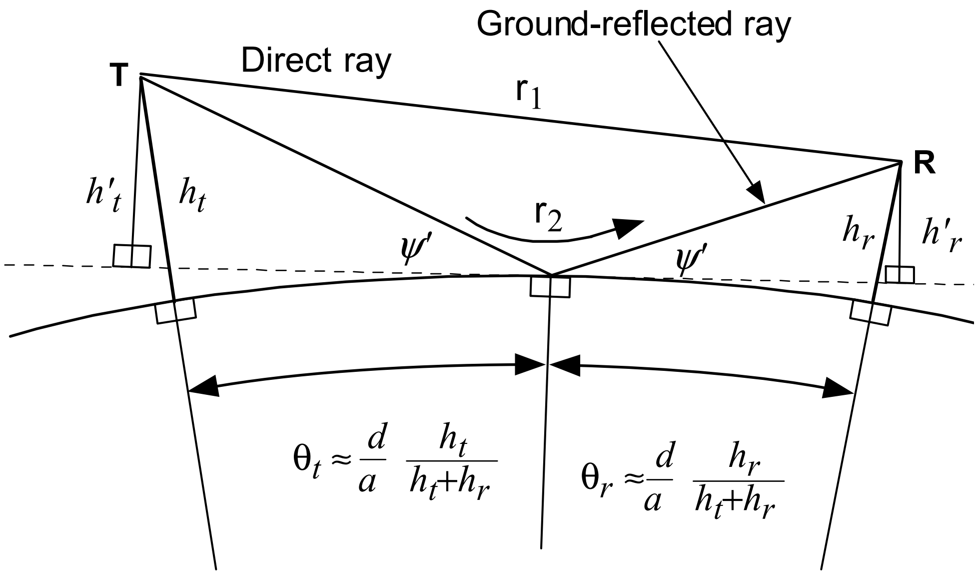
- Geometry for an LOS situation:



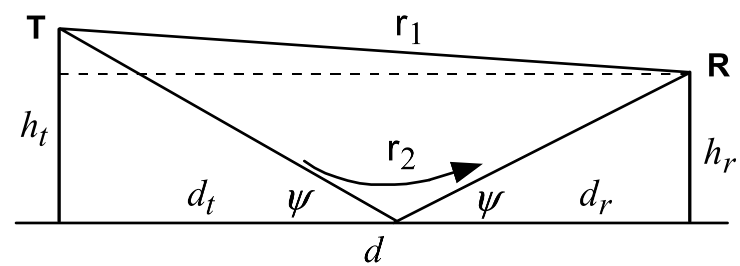
- For a given receiver antenna location, there is both a direct ray with path length r_1 and a reflected ray with path length r_2.
  - Because the orientation of the electric field is reversed when reflected (giving an apparent $180°$ phase shift for horizontally polarized waves), the reflected ray tends to act as destructive interference when $r_1 \approx r_2$, which occurs for lower antenna heights and longer distances.

Propagation in the LOS region (continued)

- Smooth-Earth geometry and its plane-Earth approximation:



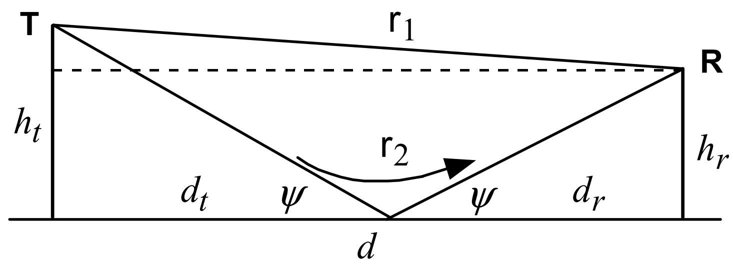
Reflection from a smooth spherical Earth



Reflection from a plane smooth Earth

- To a sufficient degree of accuracy the propagation of LOS paths can be analyzed using the plane Earth model in the right side of this figure.

Analysis of plane-Earth propagation



- The difference in path length for the direct and reflected rays is

$$r_2 - r_1 = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$\approx d\left[1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right] - d\left[1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right] = \frac{2h_t h_r}{d}$$

- This path-length difference gives rise to the phase difference

$$\Delta = \frac{4\pi h_t h_r}{\lambda d} = 1.3343\pi \times 10^{-5}\, \frac{f_{\mathrm{MHz}} h_t h_r}{d_{\mathrm{km}}}$$

when the antenna heights are expressed in meters.

Analysis of plane-Earth propagation (continued)

- The grazing angle or angle of reflection, ψ, is given by

$$\tan\psi = \frac{h_t}{d_t} = h_t \div \frac{h_t\, d}{h_t + h_r} = \frac{h_t + h_r}{d}$$

- Ignoring a component of the signal that reaches the receiver by groundwave propagation from the point of reflection that is negligible at VHF and UHF, the squared magnitude of the ratio between received and free-space electric field intensities is

$$\left|\frac{E_{rec}}{E_{fs}}\right|^2 \approx \left|1 + \mathrm{R}\, \mathrm{e}^{j\Delta}\right|^2 = 1 + |\mathrm{R}|^2 + 2|\mathrm{R}|\cos(\Delta + \xi)$$

where the complex reflection coefficient R is related to the grazing angle and the ground impedance z by

$$\mathrm{R} = \frac{\sin\psi - z}{\sin\psi + z} \equiv |\mathrm{R}|\mathrm{e}^{j\xi}$$

Analysis of plane-Earth propagation (continued)

- The ground impedance z is related to ϵ_g, the complex dielectric constant of the partially conducting Earth and is given by

$$z = \begin{cases} \sqrt{\epsilon_g - \cos^2\psi} \approx \sqrt{\epsilon_g - 1}, & \text{horizontal polarization} \\ \sqrt{\epsilon_g - \cos^2\psi}/\epsilon_g \approx \sqrt{\epsilon_g - 1}/\epsilon_g, & \text{vertical polarization} \end{cases}$$

- The value of ϵ_g is related to the free-space permittivity ϵ_{fs} as well as to ϵ and σ, the relative permittivity and conductivity of the ground, respectively, by

$$\epsilon_g = \epsilon - j\,\frac{\sigma}{2\pi f \epsilon_{fs}} = \epsilon - 1.796 \times 10^4\,\frac{j\sigma}{f_{\mathrm{MHz}}}$$

- Typical parameter values are $\sigma = 0.005$ and $\epsilon = 15$, giving $\epsilon_g = 15 - j90/f_{\mathrm{MHz}}$.
  - For $f = 100$ MHz, the resulting value of z is $3.75\angle{-1.84}°$ for horizontal polarization and $0.25\angle 1.59°$ for vertical polarization.
  - For $f = 1{,}000$ MHz, z is $3.74\angle{-0.18}°$ for horizontal polarization and $0.25\angle 0.16°$ for vertical polarization.

Analysis of plane-Earth propagation (continued)

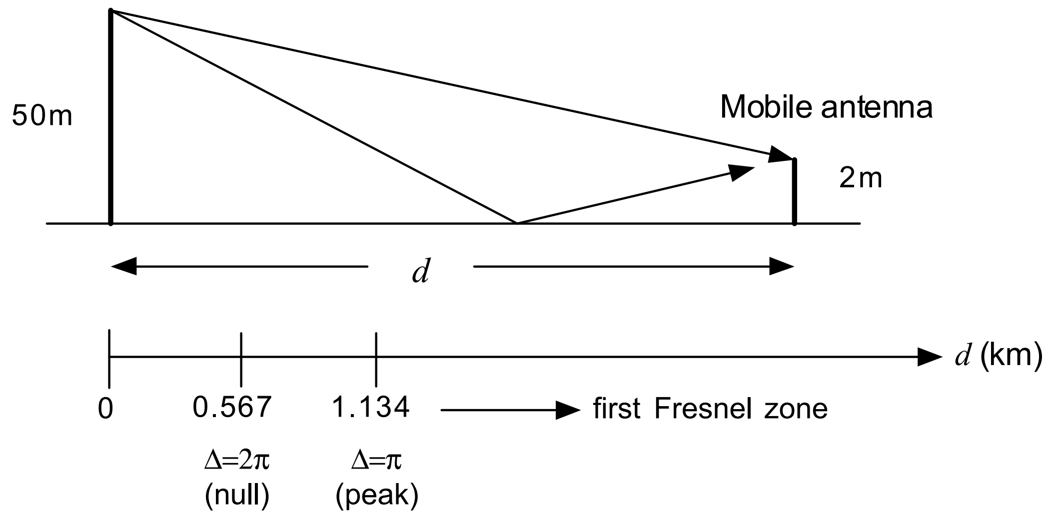
- Thus, for UHF cellular frequencies and very low grazing angles ($\psi \approx 0$), R ≈ -1 and ratio of the received power to that of the direct path only becomes

$$\left|\frac{E_{rec}}{E_{fs}}\right|^2 \approx 2(1 - \cos\Delta) = 4\sin^2(\Delta/2) = 4\sin^2\left(\frac{2\pi h_t h_r}{\lambda d}\right)$$

- This relation indicates that LOS propagation over a smooth Earth results in
  - A 6-dB signal power **gain** over free space when $\Delta/2$ is an odd multiple of $\pi/2$, when $\sin(\Delta/2) = \pm 1$
  - Signal **cancellation** when $\Delta/2$ is a multiple of π, when $\sin(\Delta/2) = 0$.
  - A variation in this angle can be due to a variation in antenna heights, link distance, or both.

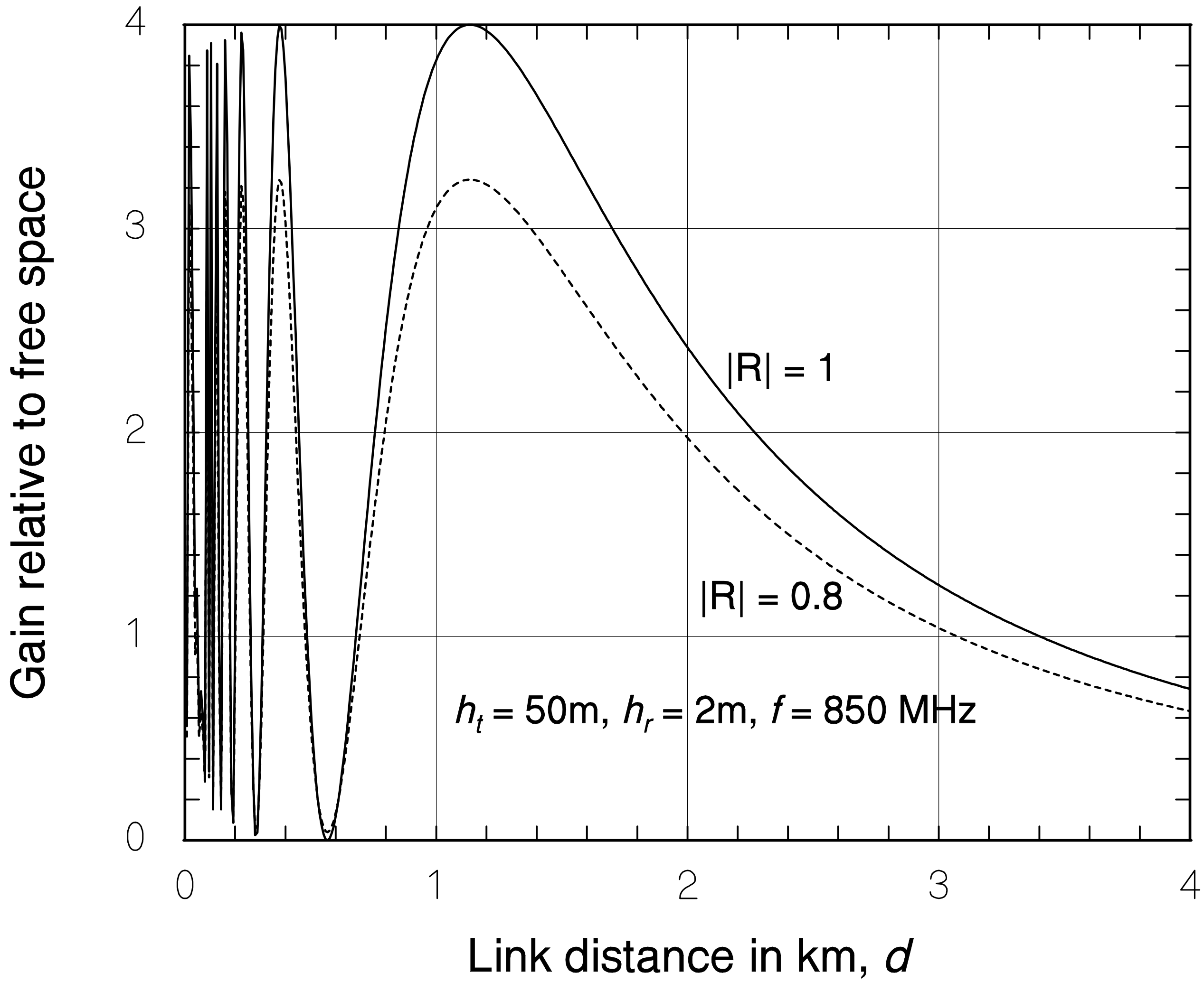
Example of plane-Earth propagation calculations

- Consider a base station antenna height $h_t = 50$ m and a mobile antenna height of $h_r = 2$ m. If the frequency is 850 MHz, then $\Delta = 1.134\pi/d_{\text{km}}$:



- For $d < 1.134$ km, the angle $\Delta/2$ is greater than $\pi/2$, and the gain, $4\sin^2(\Delta/2)$, oscillates as the mobile moves toward the base station.

- For $d > 1.134$ km, the angle Δ/2 is always less than $\pi/2$, and there are no oscillations in the attenuation as the mobile moves farther away from the base station.

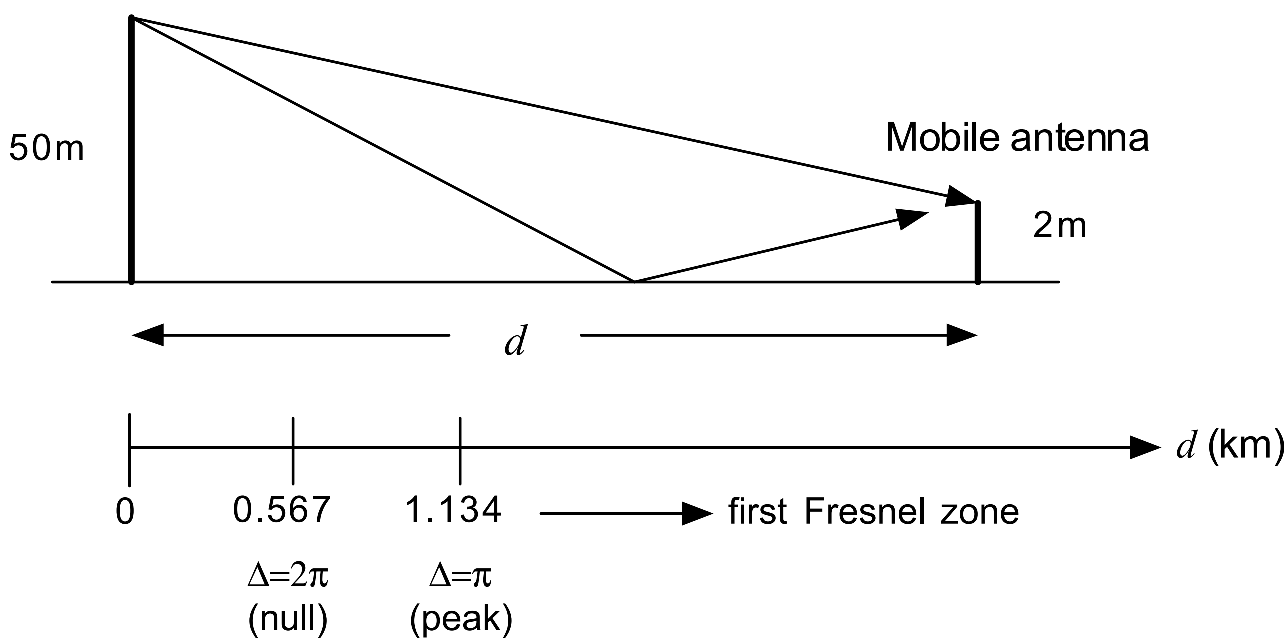

Gain relative to free space
Link distance in km, d
|R| = 1
|R| = 0.8
h_t = 50m, h_r = 2m, f = 850 MHz
0
1
2
3
4

Fresnel zones for the example

- The distance at which $\Delta = n\pi$ is given by

$$d_n = 4h_t h_r / n\lambda \quad \Leftrightarrow \quad \Delta = n\pi$$

- The set of positions for which the condition $\Delta < n\pi$ holds is referred to as the nth Fresnel zone; the first Fresnel zone corresponds to $d > d_1$.



Total path loss for the example

- The net gain for the path is obtained by multiplying by the free-space loss:

$$\frac{\mathsf{P}_r}{\mathsf{P}_t} = 4\,\mathsf{sin}^2\left(\frac{2\pi h_t h_r}{\lambda d}\right) \times \left(\frac{\lambda}{4\pi d}\right)^2$$

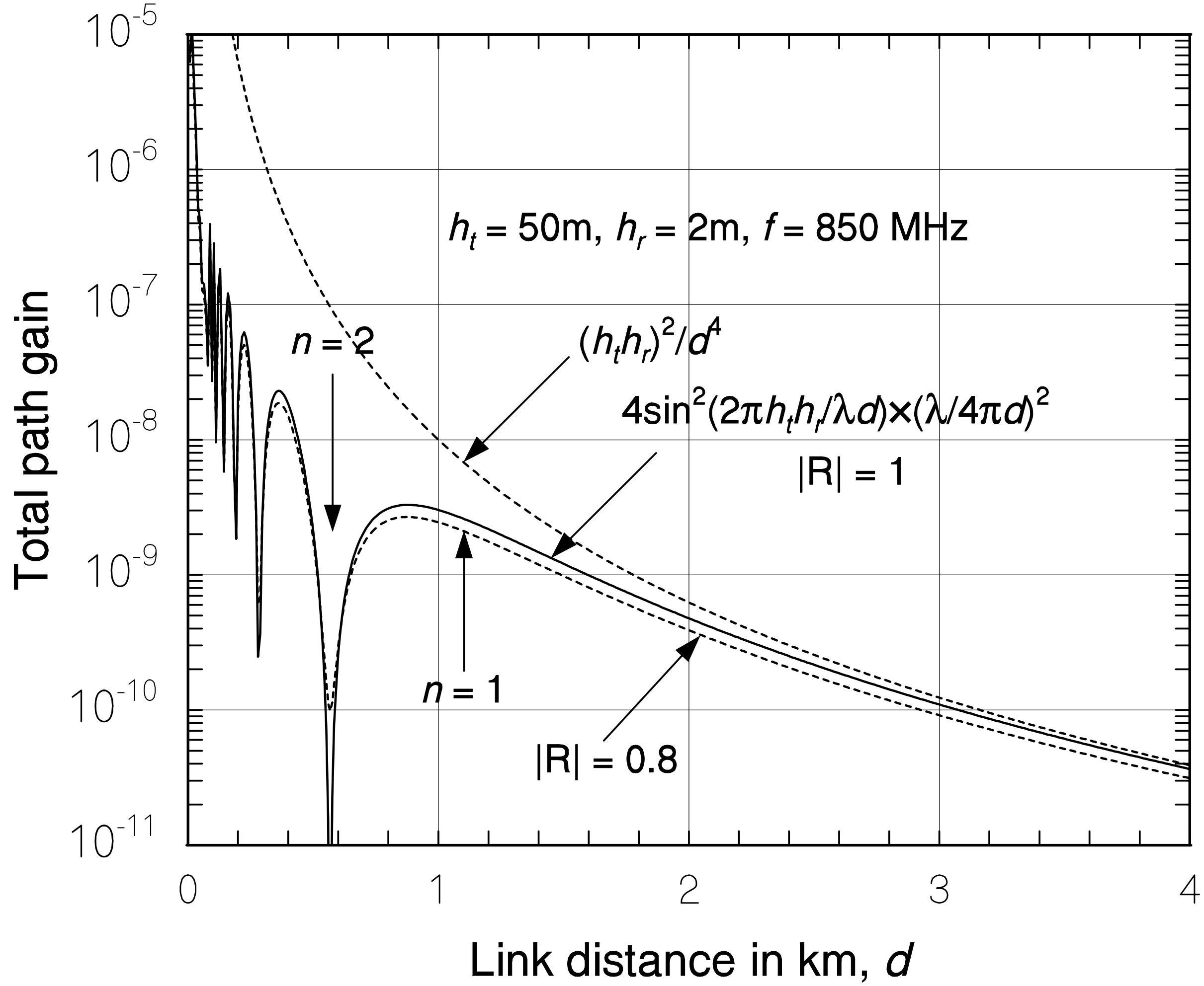
$$\approx 4\left(\frac{2\pi h_t h_r}{\lambda d}\right)^2 \left(\frac{\lambda}{4\pi d}\right)^2 = \frac{(h_t h_r)^2}{d^4},\;\; d > d'$$

where $$d' \triangleq 12 h_t h_r / \lambda$$

is the distance for which $\sin(\Delta/2) = 0.5$.

- This result shows a fourth-power dependence on distance for $d > d'$. In dB units, this LOS propagation loss is

$$L_{LOS} = 10\,\mathsf{log}_{10}\left[\frac{(h_t h_r)^2}{d^4}\right] = 120 + 40\,\mathsf{log}_{10} d_{\mathsf{km}} - 20\,\mathsf{log}_{10}(h_{t\mathsf{m}} h_{r\mathsf{m}})$$

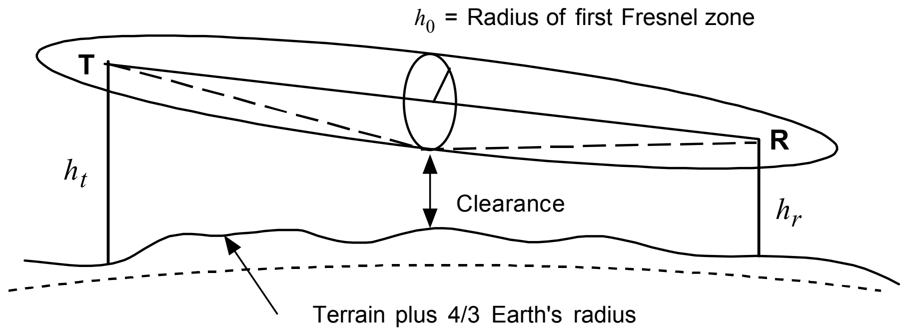


h_t = 50m, h_r = 2m, f = 850 MHz
n = 2
$(h_t h_r)^2/d^4$
$4\sin^2(2\pi h_t h_r/\lambda d)\times(\lambda/4\pi d)^2$
|R| = 1
n = 1
|R| = 0.8
Total path gain
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
0
1
2
3
4
Link distance in km, d

Fresnel zones for LOS links

- Using an example, it has been shown that the ground reflection points are within the first Fresnel zone when the mobile is relatively far from the base station.

- In general, the first Fresnel zone is defined as the elliptical volume containing reflection points for which the difference in path lengths of direct and reflected rays is less than half a wavelength:

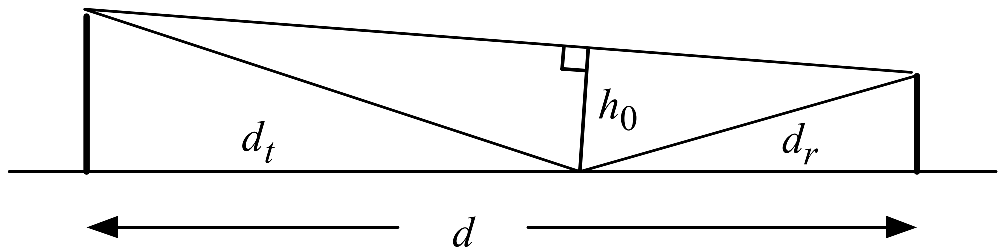


First Fresnel zone and path clearance

- The volume is elliptical because an ellipse is the locus of points for which the combined distance from two focal points (here, the length of the reflected path from transmitter to receiver) is a constant.

Fresnel zone radius

- The radius h_0 of the first Fresnel zone at the some point along a path of length d that is distance d_t from the transmitter and distance d_r from the receiver is found by solving the following equation for h_0:



$$\lambda/2 = \text{length of reflected path} - d$$

$$= \text{distance from transmitter to reflection point}$$
$$+ \text{distance from reflection point to receiver} - d$$

$$= \sqrt{d_t^2 + h_0^2} + \sqrt{d_r^2 + h_0^2} - d = d_t\left[1 + \frac{h_0^2}{2d_t^2} + \cdots\right] + d_r\left[1 + \frac{h_0^2}{2d_r^2} + \cdots\right] - (d_t + d_r)$$

$$\approx \frac{h_0^2}{2} \cdot \frac{d_t + d_r}{d_t d_r} \Rightarrow \; h_0(\text{m}) = \sqrt{\frac{\lambda d_t d_r}{d}} = 548\sqrt{\frac{d_{t\text{km}} d_{r\text{km}}}{d_{\text{km}} f_{\text{MHz}}}}$$

Fresnel zone example

- **Example:** Consider the radius of the first Fresnel zone midway in a $d = 5$ km path.

  - $d_t = d_r = 2.5$ km $\quad \Rightarrow \quad h_0(\text{m}) = 548 \times 2.5\sqrt{\dfrac{1}{5 \cdot f_{\text{MHz}}}} = \dfrac{612.7}{\sqrt{f_{\text{MHz}}}}$

  - The radius of the first Fresnel zone is $h_0 = 21$m for $f = 850$ MHz and $h_0 = 14$m for $f = 1{,}900$ MHz.

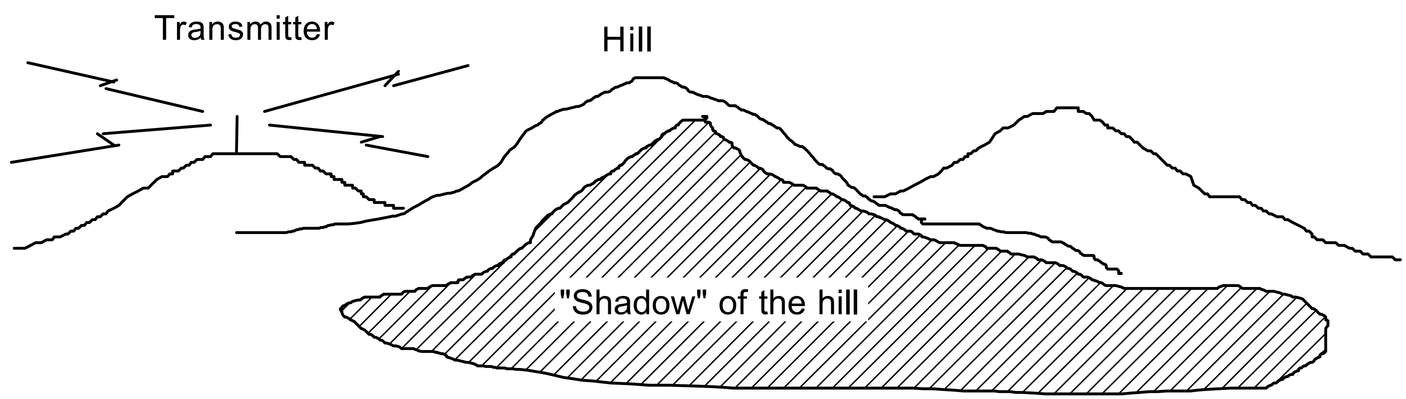
- If the direct path clears the terrain and any buildings by this amount, the reflected ray will constructively interfere with the direct ray

- Otherwise, there is the possibility of destructive multipath interference that increases with frequency.

- For this reason, it is desirable to mount the base station antenna high off the ground when possible, such as on top of a building; in an urban area, it usually is not possible to locate the antenna at a height that guarantees first Fresnel zone clearance for all potential mobile locations.

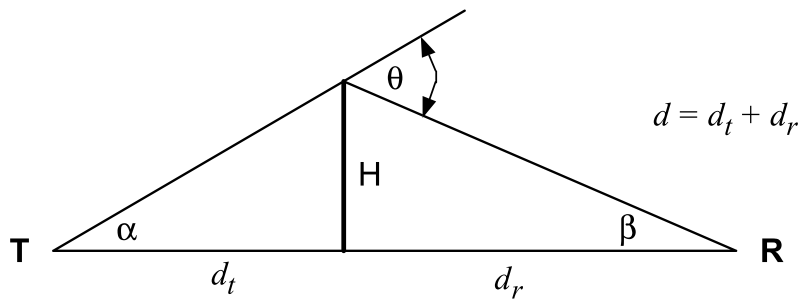
Diffraction Over Terrain and Buildings

- When the link distance d is greater than the LOS distance d_{Ls}, or when a natural or man-made obstacle blocks the direct path, there still may be a useful amount of signal power in the shadow zone at the receiver by the phenomenon of diffraction.
  - Diffraction is a fundamental property of wave motion, and in optics it is the correction to be applied to geometrical optics (ray theory) to obtain an accurate description of the way light waves bend around obstructions.
  - "The order of magnitude of the diffraction at radio frequencies may be obtained by recalling that a $1{,}000$-MHz radio wave has about the same wavelength as a $1{,}000$-Hz sound wave in air so that these two types of waves may be expected to bend around absorbing obstacles with approximately equal facility" [Bullington].



Diffraction Modeling

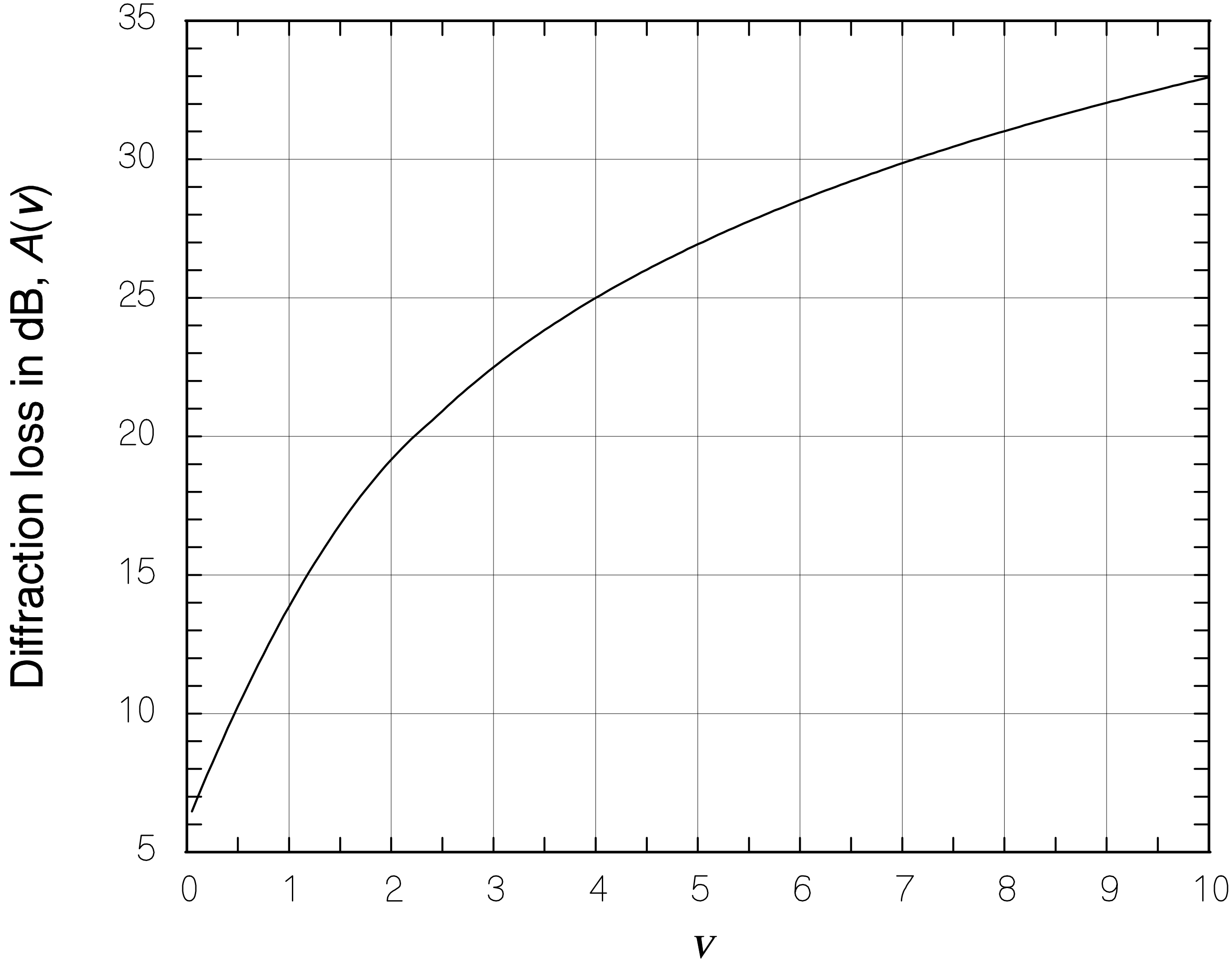
- In an urban setting, instead of a being diffracted over a hill, the signal is more likely to be diffracted over a building or row of buildings.
  - An estimate of the attenuation of the signal that results from such diffraction can be made using a “knife edge” model of the hill or building.



- In the theory of the diffraction of electric fields over a knife edge, the loss over the knife edge in dB may be approximated by

$$A(v) \approx \begin{cases} 6.02 + 9.11v - 1.27v^2, & v \leq 2.4 \\ 12.953 + 20\,\log_{10} v, & v > 2.4 \end{cases}$$

where
$$v = \sqrt{\frac{2d}{\lambda}\tan\alpha\tan\beta} = H\sqrt{\frac{2d}{d_t d_r \lambda}}$$



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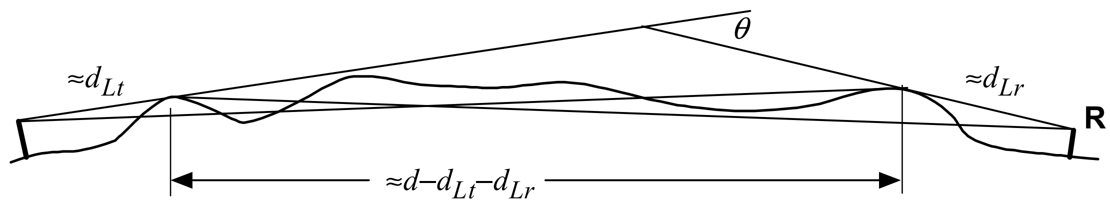
Diffraction modeling (continued)

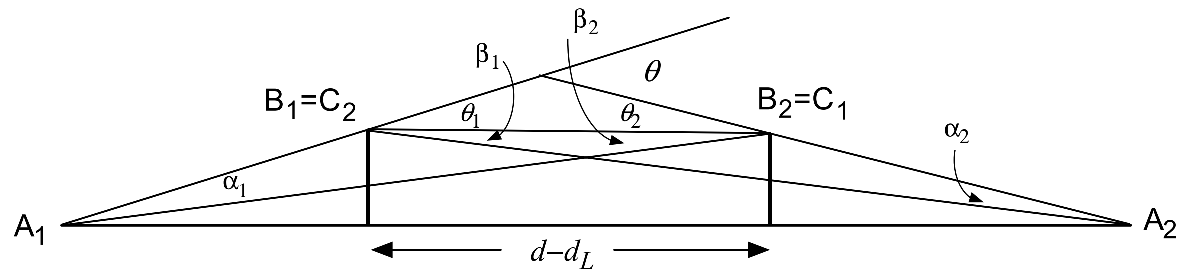
- When the path from transmitter to receiver is subject to more than one obstacle, or more than one edge, the total loss from diffraction may be estimated by adding the losses in dB of the several edges, each with its own value of the quantity v that is determined by the geometry of the situation.
  - For example, this method is used to compute the diffraction loss from different terrain or building obstacles near to both the transmitter and the receiver:



Concept of the horizons as knife edges

$$v_i^2 \approx \frac{2(d-d_L+d_{Li})}{\lambda}\,\alpha_i\,\beta_i\,,$$

$$i = 1,\,2$$



Approximate knife-edge geometry

Empirical Propagation Models

- The practicing cellular radio or systems engineer is not likely to be involved in detailed modeling of propagation paths but will use existing models and commerically available computer programs for the prediction of propagation losses and the determination of coverage areas.

- There are many mathematical models for radio propagation loss ranging from purely empirical ones, based on various measurements of loss, to semi-empirical models that use theoretical considerations to predict propagation effects based on measurements of physical parameters other than propagation loss itself.

- In general, a model is more useful if it formulates the propagation loss in terms of several parameters whose values can be made specific to the situation.

- The complexity of existing propagation models varies according to whether the calculation involves estimation of relevant physical parameters from terrain data or from field strength measurements.